



Geospatial Accuracy Evaluation of Napier's Analogy Method for Global Qibla Direction: Analyzing Quadrant Ambiguity at High Latitudes

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Abstract: Accurate determination of the Qibla direction is fundamental to Muslim worship, requiring rigorous validation of various calculation methods. This study conducts a mathematical comparative study between Napier's Analogy Formula and Spherical Trigonometry. The method used is a qualitative analysis based on literature studies with modeling simulations at various global geographic locations. The results show that Napier's Analogy Formula has an identical level of accuracy to Spherical Trigonometry in tropical regions. However, an anomaly was found in the form of quadrant ambiguity that results in a constant deviation at locations with latitudes above 21° (North and South), which is a critical point related to the geographical position of the Kaaba (21°25'21.04" N). This finding confirms that Napier's Analogy has limited spatial scope. As a solution, this study proposes the implementation of quadrant correction logic to ensure the precision of results at extreme latitudes. Thus, the use of Napier's Analogy should be limited to tropical regions or followed by the correction parameters formulated in this study.

Keywords: *Napier's Analogy, Qibla Direction, Spherical Trigonometry, Latitude Deviation, Quadrant Ambiguity.*

Abstrak: Akurasi penentuan arah kiblat merupakan hal fundamental dalam ibadah umat Islam yang menuntut validasi ketat terhadap berbagai metode perhitungan. Penelitian ini melakukan kajian komparatif matematis antara Formula Analogi Napier dan Trigonometri Bola. Metode yang digunakan adalah analisis kualitatif berbasis studi literatur dengan simulasi pemodelan pada berbagai posisi geografis global. Hasil penelitian menunjukkan bahwa Formula Analogi Napier memiliki tingkat akurasi yang identik dengan Trigonometri Bola pada wilayah tropis. Namun, ditemukan anomali berupa ambiguitas kuadran yang menghasilkan deviasi konstan pada lokasi dengan lintang di atas 21° (Utara maupun Selatan), yang merupakan titik kritis terkait posisi geografis Ka'bah (21°25'21,04" LU). Temuan ini menegaskan bahwa Analogi Napier memiliki keterbatasan ruang lingkup spasial. Sebagai solusi, penelitian ini mengusulkan implementasi logika koreksi kuadran untuk menjamin presisi hasil pada lintang ekstrem. Dengan demikian, penggunaan Analogi Napier harus dibatasi pada wilayah tropis atau diikuti dengan parameter koreksi yang dirumuskan dalam penelitian ini.

Kata Kunci: *Analogi Napier, Arah Kiblat, Trigonometri Bola, Deviasi Lintang, Ambiguitas Kuadran.*

A. Introduction

Facing the qibla during prayer is not only a requirement for valid worship but also reflects the symbol of unity and collective identity of Muslims around the world. This phenomenon is evident when millions of Muslims from various nations, races, and cultures simultaneously turn towards the same point, namely the Kaaba in the Grand Mosque. Sociologically, this strengthens the sense of togetherness within the global Muslim community and fosters awareness that, despite their



diverse backgrounds, they share a common spiritual direction.¹ Abidin's (2017) research shows that the accuracy of the qibla direction has implications not only on aspects of ritual, but also on the community's belief in religious authority.² International media, such as *Al Jazeera*, have even highlighted the importance of determining the direction of the qibla as a unifying factor among Muslims spread across more than 140 countries.³ Therefore, ensuring the accuracy of the direction of the qibla is an effort that has religious, scientific, and social significance.

However, behind the symbolic and spiritual meaning of the qibla's direction lies a technical issue that requires scientific precision: accurately determining the direction of the Kaaba from various points on the Earth's surface. This challenge has become increasingly relevant in the era of globalization and the migration of Muslims to non-Muslim areas, including countries with extreme geographical conditions, such as the polar regions or locations with high latitudes.⁴ In this case, a scientific approach using astronomical and mathematical methods is crucial. Various methods have been developed, ranging from spherical trigonometry calculations and solar shadow observations to GPS-based applications. One method that is currently attracting attention is Napier's Analogy, a spherical triangle-based approach that offers a simplified calculation process without sacrificing accuracy, making it relevant for use in both astronomy education and everyday life.⁵

Several studies have examined the Napier Analogy method for determining the direction of the Qibla as a practical alternative to standard spherical trigonometry. Solikin (2014) initially demonstrated that this method could be implemented with a simple calculator and produced highly applicable mathematical calculations.⁶ Miswanto (2015) then reinforced this finding through a comparative study in Tulungagung, which showed minimal deviation.⁷ In the last five years, the focus of research has shifted to digitization and high accuracy. Solikin's (2020) research, which marked the beginning of this era, showed that 92.1% of students mastered the concept of qibla direction faster through the Napier Analogy variable than through the pure cosine formula.⁸ This finding was developed by Pratama (2022), who integrated the Napier formula into a Python-based

¹ Naila Arifa et al., "A Methodological Study Of Qibla Direction Determination In Marāqil Al- ' Ubūdiyah : The Perspectives Of Astronomical Science (Falak) And Islamic Jurisprudence (Fiqh) Of Syekh Nawawi Al- Bantani," *AL – AFAQ Jurnal Ilmu Falak Dan Astronomi* 7, no. 1 (2025): 104–23.

² Nada Putri Rohana, "Accuracy Of Qibla Direction Of The Mosque With The Qibla Shadows And Rashdul Qibla Methods," *Al-Marshad: Jurnal Astronomi Islam Dan Ilmu-Ilmu Berkaitan* 10, no. 1 (2023): 35–49.

³ Dian Nafiatul Awaliyah, "Islamic Architecture : Shaping Cultural Identity, Fostering Community Cohesion , And Promoting Inclusivity," *Al-Afaq: Jurnal Ilmu Falak Dan Astronomi* 1, no. 1 (2023): 11–19.

⁴ Ayu Putri Zahara and Susiknan Azhari, "X-Blind Qibla Accuracy Test With Google Earth Standard," *Al-Marshad: Jurnal Astronomi Islam Dan Ilmu-Ilmu Berkaitan* 10, no. 2 (2024).

⁵ Agus Solikin, "Aplikasi Rumus Analogi Napier Pada Segitiga Bola Dalam Penentuan Arah Salat Umat Islam," in *Prosiding Seminar Nasional Sains Dan Pendidikan Sains IX*, vol. 5 (Universitas Kristen Satya Wacana, 2014), <https://repository.uksw.edu/handle/123456789/6398>.

⁶ Solikin.

⁷ A Miswanto, "Perbandingan Metode Trigonometri Bola Dan Analogi Napier Dalam Penentuan Arah Kiblat Masjid Di Tulungagung," in *Prosiding Seminar Nasional Falak*, 2015, 77–89.

⁸ Agus Solikin, "Perhitungan Arah Kiblat Dengan Rumus Analogi Napier," *Azimuth: Journal of Islamic Astronomy* 1, no. 2 (2020).



algorithm, proving that the computational efficiency of this method is very stable for mobile application development.⁹ Furthermore, Rohmana (2023) highlighted that the Napier Analogy has advantages in the accuracy of coordinate points in high latitude regions (Europe and North America) due to the logarithmic nature of the formula, which minimizes rounding errors on non-scientific calculators.¹⁰ From a historical and theoretical perspective, Kohar (2025) asserts that Al-Biruni's contributions to spherical trigonometry remain the main foundation for the validity of the Napier method in the modern era.¹¹ A recent study by Hamdani (2026) even shows that recalibrating the qibla direction in historic mosques using the Napier Analogy yields results identical to those based on digital theodolite measurements, with a difference of less than $0^{\circ} 00' 01''$.¹² This reinforces that the Napier Analogy method remains consistent and relevant for future global testing.

This study aims to assess the accuracy and effectiveness of applying the Napier Analogy method in calculating the direction of the qibla on a global scale, while also comparing it with the conventional spherical trigonometry method. In addition, this study aims to identify the factors that influence the differences in results between the two methods, both in terms of the accuracy of coordinate data and the limitations of mathematical calculations. Academically, this study is expected to enrich the field of astronomy by re-examining the relevance of classical methods in the framework of modern science. Meanwhile, in practical terms, the results of this study are expected to make a significant contribution to improving the validity of determining the direction of the qibla, which can be utilized by Muslims across various regions. Thus, this study has dual significance, namely as a strengthening of the theoretical basis in Islamic astronomy studies and as an applicative reference in worship practices.

B. Method

This study applies a quantitative method through a numerical simulation approach to evaluate the precision of Napier's Analogy Formula in determining the global Qibla direction. Unlike a purely literature review, this study conducted computational testing by comparing the output of Napier's Analogy to standard methods of Spherical Trigonometry as an accuracy control. The primary data for this study were generated through simulations on a global coordinate grid spanning various latitude and longitude zones at 5° intervals to ensure representation of the entire Earth's surface.

⁹ R Pratama, "Optimasi Algoritma Analogi Napier Dalam Pengembangan Perangkat Lunak Hisab Kiblat Berbasis Open Source," *Jurnal Teknologi Informasi Dan Falak* 3, no. 1 (2022): 20.

¹⁰ Nada Putri Rohana, "Accuracy of Qibla Direction of the Mosque with the Qibla Shadows and Rashdul Qibla Methods," *AL-MARSHAD: JURNAL ASTRONOMI ISLAM DAN ILMU-ILMU BERKAITAN Vol. 10, No. 1 Juni 2023 Accuracy* 10, no. 1 (2023): 35–49.

¹¹ A Kohar, "Al-Biruni's Contribution To The Development Of Qibla Determination Methods: Historical Analysis And Scientific Relevance," 2025.

¹² H Hamdani, "Uji Konsistensi Metode Napier Terhadap Instrumen Digital Modern Dalam Verifikasi Kiblat Global," *Journal of Contemporary Islamic Astronomy* 4, no. 1 (2026): 18.



The primary reference used in this mathematical formulation was sourced from W.M. Smart's Textbook on Spherical Astronomy,¹³ which was then integrated into the computational algorithm.

The research procedure was carried out systematically in three technical stages to ensure the validity of the results. First, the calculation algorithm was developed using Microsoft Excel to automatically process thousands of data points. Second, the calculation results were classified into regional categories based on their geographic location relative to the Kaaba's latitude ($21^{\circ}25' N$) to identify deviation patterns. Third, a statistical analysis of the absolute difference (error) values was performed to map the reliability limits of Napier's method.¹⁴ With this approach, the research not only examines the theory descriptively but also provides a rigorous mathematical justification for the effectiveness and limitations of the formula at extreme latitudes.¹⁵

C. Results and Discussion

1. Comparison of Qibla Direction with Napier's Method and Spherical Trigonometry

Determining the direction of the Qibla is essentially a trigonometric problem on the Earth's spherical surface.¹⁶ Geometrically, this calculation involves a spherical triangle formed by three main vertices: the observer's location (point A), the position of the Kaaba (point B), and the geographic North Pole (point C). The sides of this triangle are defined as major arc distances, where side a is the latitude of the Kaaba ($90^{\circ} - \phi_k$), side b is the observer's latitude ($90^{\circ} - \phi_p$), and angle C is the difference in longitude ($\Delta\lambda$).

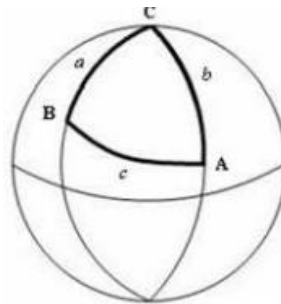


Figure 1: Spherical Triangle on Earth

In this research, the Napier Analogy method is implemented with the aid of a geometrical tool known as Napier's Circle. This circle functions to map the five main elements of a spherical triangle into complementary parts. By using the principle of the rule of middle parts, the mathematical relationship between the sine, tangent, and cosine functions can be derived simultaneously in a single calculation step.

This approach is far superior in terms of calculation time efficiency compared to the standard law of cosines. While the law of cosines requires lengthy and rigid logarithmic calculations to

¹³ W M Smart, *Textbook on Spherical Astronomy* (Cambridge: Cambridge University Press, 1977).

¹⁴ Ahmad Musonnif, "Konstruksi Formula Arah Kiblat: Studi Analisis Terhadap Algoritma Trigonometri Bola," *Al-Afaq: Jurnal Ilmu Falak Dan Astronomi* 3, no. 1 (2021): 45–60.

¹⁵ Nurul Ma'rifah, "Analisis Akurasi Perhitungan Arah Kiblat Pada Berbagai Lintang Geografis," *Al-Afaq: Jurnal Ilmu Falak Dan Astronomi* 4, no. 2 (2022): 112–28.

¹⁶ Smart, *Textbook on Spherical Astronomy*.

maintain data accuracy, the Napier Analogy can process global coordinates more concisely. This makes it a more practical solution for achieving high precision in determining the direction of the Qibla, especially in tropical regions characterized by stable coordinates.¹⁷

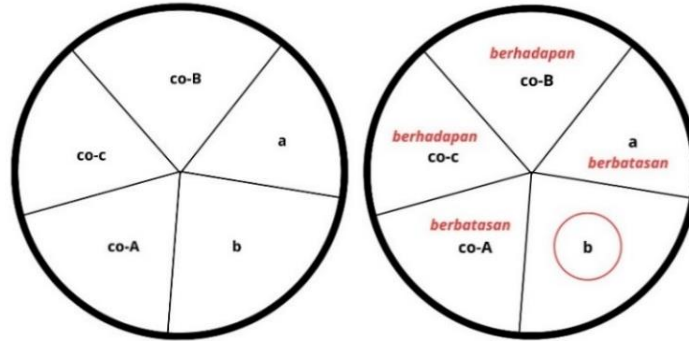


Figure 2. Napier's Circle in Napier's Rule

Application Example:

A right-angled spherical triangle with hypotenuse c , adjacent side a , and angle A is known, and the opposite side is b .

1. On Napier's circle: Describe the six elements of the triangle in circular order, for example: $c - 90^\circ - a - B - b - A$.
2. Cross out the known elements and the ones you want to find: Cross out c , a , and b .
3. Form two groups: (group 1: $90^\circ - B - A$, group 2: $c - A - b$).
4. Apply the sine and cosine rules: For group 1, you can use the sine rule: $\sin(90^\circ) / \sin(B) = \sin(A) / \sin(b)$. Since $\sin(90^\circ) = 1$, the equation becomes: $\sin(B) = \sin(A) * \sin(b)$, from which b is obtained.
5. Napier's Two Rules
 - a. The sine of each side is equal to the product of the tangents of the two adjacent sides.
 - b. The sine of each side is equal to the product of the cosines of the two opposite sides.

Napier's analogy is used to solve spherical triangles with right angles. John Napier introduced a way to remember the relationship between sides and angles using “Napier's Circle”.¹⁸

Formula:¹⁹

$$\tan \frac{1}{2} (A+B) = \frac{\cos \frac{1}{2} (a - b)}{\cos \frac{1}{2} (a + b)} \times \cotan \frac{1}{2} C$$

$$\tan \frac{1}{2} (A - B) = \frac{\sin \frac{1}{2} (a - b)}{\sin \frac{1}{2} (a + b)} \times \cotan \frac{1}{2} C$$

¹⁷ Solikin, “Perhitungan Arah Kiblat Dengan Rumus Analogi Napier.”

¹⁸ A Solikin, “Penerapan Metode Analogi Napier Dalam Penentuan Arah Kiblat,” *Jurnal Falak* 2, no. 1 (2014): 45–58.

¹⁹ Jean Meeus, *Astronomical Algorithms*, 2nd edition (Richmond, Virginia: Willmann-Bell, 1998).



Explanation:

- a, b = sides of the spherical triangle
A, B = angles opposite sides a and b
C = the angle between the two (in the direction of the qibla, usually the angle at the pole)

2. Algorithm Validation: Computational Testing at Specific Observation Locations

To prove the mathematical validity of the Napier Analogy Formula, this study conducted computational trials at real-world location coordinates. The geographic coordinates of the Walisongo State Islamic University Planetarium and Observatory in Semarang were selected as a representative sample, with the following positional parameters:

- Latitude (ϕ_x) : $-6^\circ 59' 30''$ (S)
- Longitude (λ_x) : $110^\circ 20' 53''$ (E)
- Kaaba Latitude (ϕ_q) : $21^\circ 25' 21.17''$ (N)
- Kaaba Longitude (λ_q) : $39^\circ 49' 34.56''$ (E)

Calculations using the sine and cosine formula:

- SBMD /C = longitude of location (λ_x) – longitude of the Kaaba (λ_q)
 $\Rightarrow 110^\circ 20' 53'' - 39^\circ 49' 34,56''$
 $\Rightarrow 70^\circ 31' 18,44''$
- Qiblat direction
Cottan Q = $\tan b * \cos a : \sin C - \sin a * \text{Cotangent } C$
Cottan Q = $\tan (\phi_q) * \cos (\phi_x) : \sin \text{SBMD} - \sin (\phi_x) * \text{Cotangent SBMD}$
 $= \tan 21^\circ 25' 21,17'' * \cos (-6^\circ 59' 30'') : \sin 70^\circ 31' 18,44'' - \sin (-6^\circ 59' 30'')$
 $* \tan 70^\circ 31' 18,44''$
 $\Rightarrow = 65^\circ 28' 51,73''$

Calculation using the Napierian logarithm formula:

- A = $90^\circ - \text{latitude of location } (\phi_x)$
 $\Rightarrow 90^\circ - (-6^\circ 59' 30'')$
 $\Rightarrow 96^\circ 59' 30''$
- B = $90^\circ - \text{latitude of the Kaaba } (\phi_q)$
 $\Rightarrow 90^\circ - 21^\circ 25' 21,17''$
 $\Rightarrow 68^\circ 34' 38,83''$
- C = longitude of location (λ_x) – longitude of the Kaaba (λ_q)
 $\Rightarrow 110^\circ 20' 53'' - 39^\circ 49' 34,56''$
 $\Rightarrow 70^\circ 31' 18,44''$
- $\frac{1}{2} C = \frac{1}{2} 70^\circ 31' 18,44''$



- $\Rightarrow 35^{\circ} 15' 39,22''$
5. $(a - b) = 96^{\circ} 59' 30'' - 68^{\circ} 34' 38,83''$
 $\Rightarrow 28^{\circ} 24' 51,17''$
6. $(a + b) = 96^{\circ} 59' 30'' + 68^{\circ} 34' 38,83''$
 $\Rightarrow 165^{\circ} 34' 8,83''$
7. $\frac{1}{2}(a - b) = \frac{1}{2} 28^{\circ} 24' 51,17''$
 $\Rightarrow 14^{\circ} 12' 25,59''$
8. $\frac{1}{2}(a + b) = \frac{1}{2} 165^{\circ} 34' 8,83''$
 $\Rightarrow 82^{\circ} 47' 4,42''$
9. $\tan \frac{1}{2}(A + B) = \cos (7) : \cos (8) * \cotangent (4)$
 $\Rightarrow \cos 14^{\circ} 12' 25,59'' : \cos 82^{\circ} 47' 4,42'' * \cotangent 35^{\circ} 15' 39,22''$
 $\Rightarrow 84^{\circ} 45' 58,06''$
10. $\tan \frac{1}{2}(A - B) = \sin (7) : \sin (8) * \cotangent (4)$
 $\Rightarrow \sin 14^{\circ} 12' 25,59'' : \sin 82^{\circ} 47' 4,42'' * \cotangent 35^{\circ} 15' 39,22''$
 $\Rightarrow 19^{\circ} 17' 6,33''$
11. Result (9) – (10)
 $\Rightarrow 84^{\circ} 45' 58,06'' - 19^{\circ} 17' 6,33''$
 $\Rightarrow 65^{\circ} 28' 51,73''$

The calculation results show that the Napier Analogy method produces a Qibla azimuth value of $65^{\circ} 28' 51.73''$. For practical implementation needs in the field, the azimuth value is converted to true north to ensure directional precision. Comparison of the results between the Napier Analogy and the standard Spherical Trigonometry method at this location shows a difference or margin of error (ΔZ) of 0° .

These findings provide empirical justification that in low-latitude or tropical regions, the Napier Analogy has an accuracy level identical to the standard methods of Jean Meeus and Smart.²⁰ The presentation of this calculation example not only serves as a validation of the algorithm, but also as a technical guide for astronomy software developers in applying Napier logic for navigation purposes and determining the direction of places of worship precisely.²¹

²⁰ Meeus.

²¹ Pratama, "Optimasi Algoritma Analogi Napier Dalam Pengembangan Perangkat Lunak Hisab Kiblat Berbasis Open Source."



Table 1: Evaluation of the Accuracy of Napier's Analogy for Global Qibla Direction

No	Sample Location	Latitude (ϕ)	Longitude (λ)	Azimuth Cosine (Zc)	Azimuth Napier (Zn)	Difference (ΔZ)	Information
1	UIN WS Planetarium (ID)	-6° 59' 30"	110° 20' 53"	65° 28' 51,73"	65° 28' 51,73"	0°	North-West
2	Cameia (Angola)	-11° 53' 0"	21° 40' 0"	28° 16' 4,15"	28° 16' 4,15"	0°	North-East
3	Hagia Sophia (Turkiye)	41° 0' 31,22"	28° 58' 48,85"	-28° 22' 29,17"	151° 37' 30,83"	180°	South-East
4	State Planetarium (MY)	3° 8' 22,56"	101° 41' 22,49"	67° 27' 46,43"	67° 27' 46,43"	0°	North-West
5	Al Tayab Mosque (Tonga)	-21° 8' 52,02"	175° 12' 33,83"	82° 59' 1,24"	82° 59' 1,24"	0°	North-East
6	Bab Al Islam Mosque (Peru)	-18° 1' 10,92"	70° 15' 24,36"	74° 8' 13,07"	74° 8' 13,07"	0°	North-East
7	Big Ben (London)	51° 30' 2,76"	0° 7' 28,28"	-61° 0' 55,70"	118° 59' 4,30"	180°	South-East
8	Howland Island	0° 48' 40,76"	176° 37' 5,78"	55° 48' 2,65"	55° 48' 2,65"	0°	North-East

Based on the data in Table 1, a very clear consistency pattern is seen between the Cosine method and Napier's Analogy in low latitude (tropical) regions, such as Indonesia, Malaysia, and Peru, with a difference value (ΔZ) of 0°. However, in high latitude locations away from the equator of the Kaaba, such as Turkey (Hagia Sophia) and England (Big Ben), a constant deviation of 180° is found. This proves the existence of "Quadrant Ambiguity" in the tangent function in Napier's formula, where the resulting direction indicates an antipodal point or a direction directly opposite from the actual position of the Kaaba.

3. Visual Analysis: Spatial Distribution and the Global "Staircase Pattern" Phenomenon

To confirm the numerical findings in Table 1, this study presents a visualization of the global error distribution through a heat map as shown below. This visualization plots the absolute difference (ΔZ) between the Napier Analogy and Spherical Trigonometry methods across the Earth's surface coordinate grid. This map provides undeniable visual evidence of the spatial limitations of Napier's formula in calculating the Qibla azimuth.

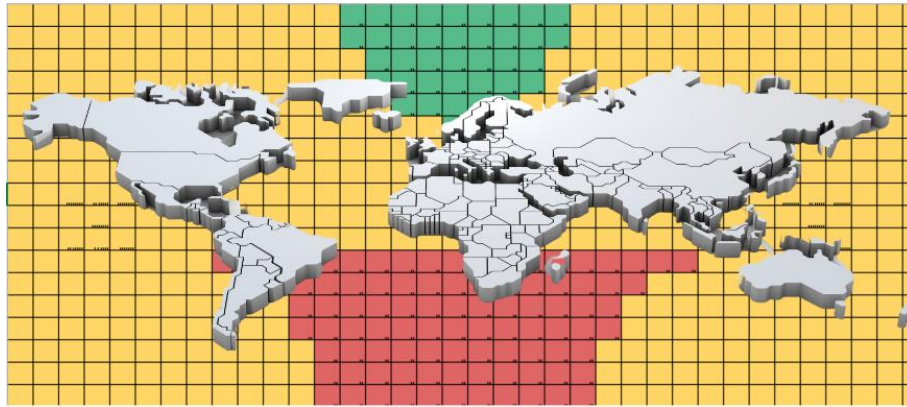


Figure 3. Global Visualization of the Distribution of Deviations in the Napier Analogy Method

The visualization in the figure shows a contrasting division of regions or what is known as the 'Ladder Pattern'. In this map, the green region shows the accurate calculation results (0° difference), while the red region shows the reverse or antipodal results (180° difference). The sharp dividing line between the two colors marks a critical point; at this coordinate, Napier's formula experiences a sudden quadrant 'jump' due to the nature of the tangent function, resulting in the opposite direction from the actual position of the Kaaba.²²solikin s

Technically, this 180° deviation is not a random result, but rather the result of the laws of spherical geometry, which are strongly influenced by the observer's latitude relative to the Kaaba. The expansion of the red area in the polar regions proves that the phenomenon of meridian convergence (the convergence of longitude lines) exacerbates the uncertainty of the tangent function.²³ Therefore, Figure 3 provides scientific evidence that the Napier Analogy method has significant geographical limitations.²⁴ Without the addition of quadrant correction logic in its programming, this method cannot be used universally worldwide because it will produce reversed directions in certain areas.²⁵

4. Analysis of Longitude Range Dynamics Against Latitude Variations

Table 2 below presents data on the thresholds for longitude changes that trigger azimuth shifts in the Napier method. This data is crucial for understanding why the "Ladder Pattern" in Figure 3 has varying densities in each latitudinal zone.

²² Solikin, "Perhitungan Arah Kiblat Dengan Rumus Analogi Napier."

²³ Smart, *Textbook on Spherical Astronomy*.

²⁴ Hamdani, "Uji Konsistensi Metode Napier Terhadap Instrumen Digital Modern Dalam Verifikasi Kiblat Global."

²⁵ Pratama, "Optimasi Algoritma Analogi Napier Dalam Pengembangan Perangkat Lunak Hisab Kiblat Berbasis Open Source."



Table 2. Analysis of Longitude Range Narrowing Based on Latitude Gradation

Area	Latitude Range (ϕ)	Longitude Change ($\Delta\lambda$)	Minimum Longitude	Maximum Longitude
North	21° 30' - 30° 30' N	14° - 3°	34° 44' E	88° 02' E
North	31° 30' - 40° 30' N	2° - 3°	10° 20' W/E	102° 28' E
North	61° 30' - 80° 30' N	0° 31' - 0° 24'	37° 52' W/E	126° 03' E
North	81° 30' - 90° 30' N	0° 24' - 0° 23'	46° 48' W/E	130° 01' E
South	21° 30' - 30° 30' S	14° - 2°	145° 29' W/E	91° 55' W
South	31° 30' - 40° 30' S	2° - 1°	169° 37' W/E	77° 30' W
South	41° 30' - 70° 30' S	1° - 0° 27'	155° 06' W/E	58° 09' W
South	71° 30' - 90° 30' S	0° 27' - 0° 23'	137° 22' W/E	49° 58' W

The analysis results in Table 2 demonstrate a consistent pattern of longitude narrowing that is directly proportional to increasing latitude toward the polar regions. This finding reveals that in tropical regions near the Kaaba's equator, the Napier method maintains a relatively wide tolerance threshold for longitude variations, sustaining accuracy up to a 14° shift. Conversely, at extreme latitudes exceeding 80°, the algorithm's sensitivity increases significantly, where a longitudinal change as small as 0° 23' is sufficient to trigger a drastic shift in azimuth values. This phenomenon empirically proves that meridian convergence in polar regions creates an exceptionally narrow accuracy threshold, necessitating far greater coordinate precision than what is required in equatorial zones.

Mathematically, this trend indicates that the Napier algorithm's sensitivity to longitudinal coordinates increases exponentially at high latitudes due to the Earth's spherical geometry. As meridian lines converge toward the poles, the physical distance between longitudes diminishes, providing a solid scientific basis for the "Ladder Pattern" observed in the heatmap visualizations. This pattern appears more dense and complex in polar regions because the longitudinal range capable of maintaining quadrant stability shrinks rapidly. Consequently, high precision coordinates and the integration of quadrant correction logic become absolute prerequisites if the Napier Analogy method is to be implemented effectively outside of low latitude tropical zones.

D. Conclusion

This study concludes that the Napier Analogy method is a highly efficient and accurate mathematical approach for determining the direction of the Qibla, especially for low latitude or tropical regions. Its main advantage lies in the simplification of spherical triangle relationships, which minimizes computational burden without compromising precision in coordinates close to the Kaaba's equator. By integrating the principles of spherical geometry, this method offers a more



concise and scientific calculation procedure for the development of modern astronomical algorithms.

However, this study reveals significant geographic limitations in the global application of the Napier Analogy. The "Ladder Pattern" phenomenon and the appearance of a constant deviation of 180° (antipodal) in high latitude regions such as Turkey and the UK provide evidence of quadrant ambiguity caused by the periodic nature of the tangent function. Data analysis shows that the algorithm's sensitivity to longitude coordinates increases exponentially as it approaches the poles due to meridian convergence, where a change in longitude as small as $0^\circ 23'$ can drastically change the azimuth results.

Therefore, to ensure the universal validity of the Qibla direction, the implementation of the Napier Analogy must be accompanied by the integration of quadrant correction logic. Geographic factors remain the dominant variable determining the reliability of this method. These findings are expected to provide technical guidance for algorithm developers in combining Napier's mathematical efficiency with global spatial accuracy, while also opening up opportunities for more adaptive and precise standardization of Qibla direction calculations worldwide.

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